HSS BRACE CONNECTIONS TO HSS COLUMNS AND WF BEAMS
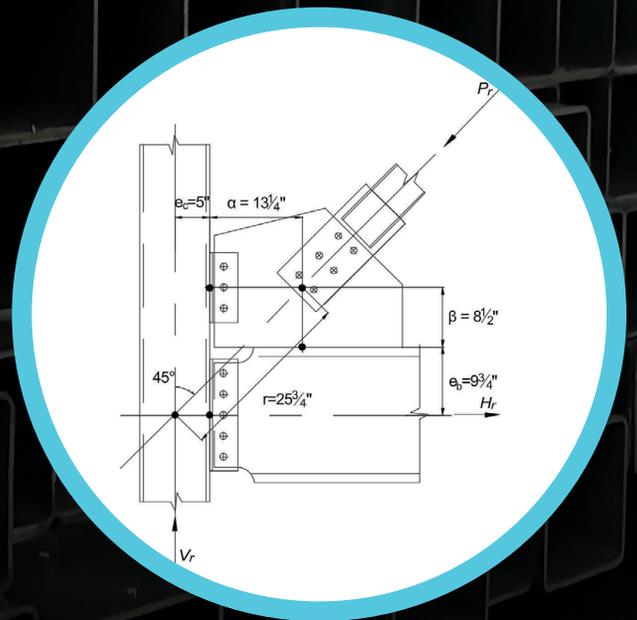
by Jeffrey Packer, Ph.D., D.Sc., P.Eng. and Sam Richardson, MASc Candidate

HSS Brace to HSS Column to Wide-Flange Beam Connection

by Jeffrey Packer, Ph.D., D.Sc., P.Eng., Professor of Civil Engineering, University of Toronto, ON
and Sam Richardson, M.A.Sc., Engineer, Canadian Institute of Steel Construction, Toronto, ON

_____

A beam-to-column connection in a braced frame is shown in Figure 1, with selected members subject to the factored loads indicated. A connection design will be performed using Group 150 7/8" bolts with threads included in the shear plane (thread condition N), with standard holes in the beam web, plates, and angles. The HSS column and brace are ASTM A500/A500M Grade C, the beam is ASTM A992/A992M, the four clip angles and the plates are ASTM A572/A572M Grade 50. Weld electrodes are 70 ksi.

Fig. 1. HSS brace to HSS column to wide-flange beam connection

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Column and Brace
ASTM A500/A500M Grade C
$F_y = 50$ ksi
$F_u = 62$ ksi

Beam
ASTM A992/A992M
$F_y = 50$ ksi
$F_u = 65$ ksi

Clip Angles
ASTM A572/A572M Grade 50
$F_y = 50$ ksi
$F_u = 65$ ksi

Plates
ASTM A572/A572M Grade 50
$F_y = 50$ ksi
$F_u = 65$ ksi

1

From AISC *Manual* Tables 1-1 and 1-12, the geometric properties are as follows:

Beam	Column	Brace
W18×143	HSS10×10×1/2	HSS6×6×3/8
$t_w = 0.730$ in.	$B = 10.0$ in.	$B = 6.0$ in.
$d = 19\text{-}1/2$ in.	$H = 10.0$ in.	$H = 6.0$ in.
$T = 15\text{-}1/8$ in.	$t = 0.465$ in.	$t = 0.349$ in.
	$A = 17.2$ in.^{2}	$A = 7.58$ in.^{2}

Force Transfer in Diagonal Bracing Connections: The Uniform Force Method (Manual Part 13)

The distance from the face of the column flange to the ideal centroid of the gusset-to-beam connection is (by First Moments of Area):

$$\alpha = \frac{(27 \text{ in.})^2(20 \text{ in.})(1/2) - (16 \text{ in.})(4 \text{ in.})(1/2)(16 \text{ in.})(1/3) - (11 \text{ in.})^2(1/2)[16 \text{ in.} + (11 \text{ in.})(2/3)]}{(27 \text{ in.})(20 \text{ in.}) - (16 \text{ in.})(4 \text{ in.})(1/2) - (11 \text{ in.})^2(1/2)} + 0.5 \text{ in.}$$

$$= 13\text{-}1/4 \text{ in.}$$

The distance from the face of the beam top flange to the ideal centroid of the gusset-to-beam connection is (by First Moments of Area):

$$\beta = \frac{(27 \text{ in.})(20 \text{ in.})(20 \text{ in.})(1/2) - (16 \text{ in.})(4 \text{ in.})(1/2)[16 \text{ in.} + (4 \text{ in.})(2/3)] - (11 \text{ in.})^2(1/2)[9 \text{ in.} + 11 \text{ in.}(2/3)]}{(27 \text{ in.})(20 \text{ in.}) - (16 \text{ in.})(4 \text{ in.})(1/2) - (11 \text{ in.})^2(1/2)}$$

$$= 8\text{-}1/2 \text{ in.}$$

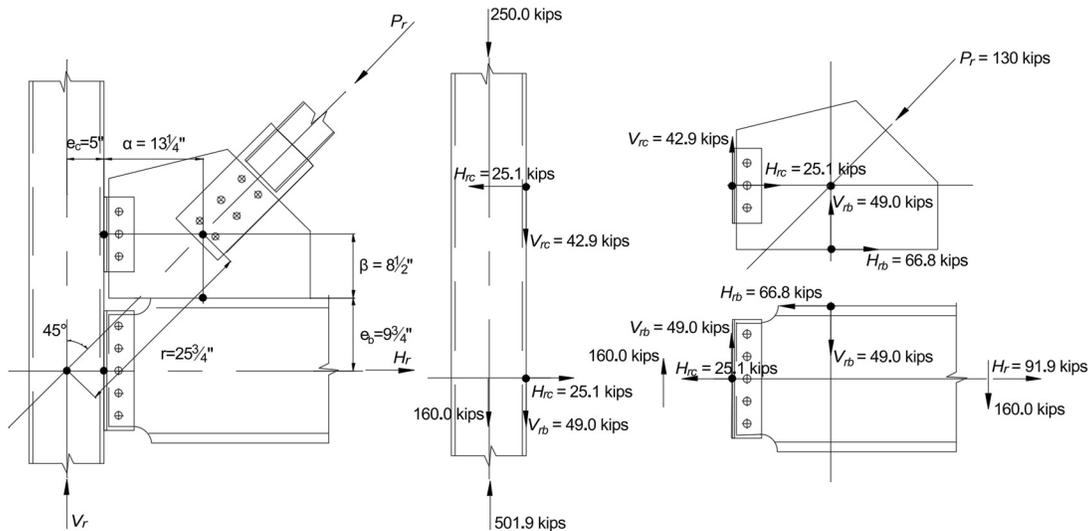To use the uniform force method, the geometry in Figure 2-a must satisfy the following expression:

$$\alpha - \beta \tan(\theta) = e_b \tan(\theta) - e_c \qquad \text{(from *Manual* Eq. 13-1)}$$

$e_b = 9.75$ in.   (one-half the depth of the beam)
$e_c = 5$ in.   (one-half the depth of the column)
$\theta = 45°$ (angle between the brace axis and vertical)

13-1/4 in. $-$(8-1/2 in.)$\tan(45°) = (9.75$ in.)$\tan(45°) - 5$ in.
4.75 in. $= 4.75$ in.   **o.k.**

This means that no moments act at the interfaces between the gusset plate, beam, and column (Dranger & Thornton, 2025). Therefore, the forces may be partitioned per the uniform force method. This results in the free-body diagrams for the gusset plate, beam, and column, as shown in Figure 2-b (Dranger & Thornton, 2025).

a) Connection geometry and the gusset plate centroid   b) Gusset plate, beam, and column free-body diagrams

Fig. 2. Geometry (a) and individual component free-body diagrams (b) from the Uniform Force Method

The force components are defined below, where $r = \sqrt{(\alpha + e_c)^2 + (\beta + e_b)^2} = $ 25-3/4 in. (*Manual* Eq. 13-6).

$$V_{rc} = \frac{\beta}{r}P_r \qquad \text{(from \textit{Manual} Eq. 13-2)}$$
$$= \frac{\text{8-1/2 in.}}{\text{25-3/4 in.}}(130 \text{ kips})$$
$$= 42.9 \text{ kips}$$

$$H_{rc} = \frac{e_c}{r}P_r \qquad \text{(from \textit{Manual} Eq. 13-3)}$$
$$= \frac{\text{5 in.}}{\text{25-3/4 in.}}(130 \text{ kips})$$
$$= 25.1 \text{ kips}$$

$$V_{rb} = \frac{e_b}{r}P_r \qquad \text{(from \textit{Manual} Eq. 13-4)}$$
$$= \frac{\text{9.75 in.}}{\text{25-3/4 in.}}(130 \text{ kips})$$
$$= 49.0 \text{ kips}$$

$$H_{rb} = \frac{\alpha}{r}P_r \qquad \text{(from \textit{Manual} Eq. 13-5)}$$
$$= \frac{\text{13-1/4 in.}}{\text{25-3/4 in.}}(130 \text{ kips})$$
$$= 66.8 \text{ kips}$$

Strength of the fillet welds between the HSS and splice plate

Assuming the HSS slot is 1/16 in. wider than the gusset thickness, the gap for either weld will not exceed 1/16 in., so no adjustment of the weld size is required per AWS D1.1/D1.1M. Therefore, the weld effective size is the full weld: w= 1/4 in. From AISC *Design Guide 24* Table 3-3, t_{min} = 0.200 in. for ASTM A500/A500M Grade C HSS with a 1/4 in. weld size. The design thickness of the HSS (t = 0.349 in.) is greater than t_{min} = 0.200 in. Therefore, the shear strength of the weld metal controls over the shear strength of the HSS base metal.

From AISC *Specification* Section J2.4, the available weld strength, per unit length, is:

$$\phi R_n = \phi 0.60 F_{EXX}\frac{w}{\sqrt{2}}l(4 \text{ lines}) \qquad \text{(from \textit{Spec}. Eq J2-4)}$$
$$= 0.75(0.60)(70 \text{ ksi})\frac{1/4}{\sqrt{2}}(6.00 \text{ in.})(4 \text{ lines})$$
$$= 133.6 \text{ kips} > 130 \text{ kips} \quad \textbf{o.k.}$$

From the AISC *Specifications* Table J2.4 the minimum weld size is 3/16 in. Therefore, a 1/4 in. size is sufficient.

Strength of the bolts between the splice plate and gusset plate (lap connection)

The available strength of a 7/8 in. Group 150 bolt (condition N) in single shear is given as 30.7 kips per bolt in Table 7-1 of the AISC *Manual*.

The available strength of the bolts in bearing is given by AISC *Specification* Section J3.11, assuming deformation at service load is a design consideration.

$$\phi R_n = \phi 2.4 dt F_u \qquad \text{(from \textit{Spec}. Eq. J3-6a)}$$
$$= 0.75(2.4)(7/8 \text{ in.})(3/4 \text{ in.})(65 \text{ ksi})$$
$$= 43.9 \text{ kips/bolt}$$

Therefore, the strength of the bolt group is governed by bolt shear.

$$(6 \text{ bolts})(30.7 \text{ kips/bolt}) = 184.2 \text{ kips} > 130 \text{ kips} \quad \textbf{o.k.}$$

Bolt spacing requirements:

Minimum edge distance $= 1\text{-}1/8 \text{ in.} < 2 \text{ in.}$ **o.k.** (from *Spec*. Table J3.4)
Maximum edge distance $= 12t_p = 12(3/4 \text{ in.}) = 9 \text{ in.} > 2 \text{ in.}$ **o.k.** (from *Spec*. J3.6)
Minimum spacing $= 2\text{-}2/3d = 2\text{-}2/3(7/8 \text{ in.}) = 2.33 \text{ in.} < 4 \text{ in.}$ **o.k.** (from *Spec*. J3.4)
Maximum spacing $= 24t_p = 24(3/4 \text{ in.}) = 18 \text{ in.} > 4 \text{ in.}$ **o.k.** (from *Spec*. J3.6a)

Buckling of the gusset plate

From AISC *Manual* Part 9, the width of the Whitmore section is:

$l_w = 3 \text{ in.} + 2[2(4 \text{ in.})\tan 30°]$ (from *Manual*. Fig. 9-1a)

$\quad = 12.24 \text{ in.}$

The splice plate is eccentric to the gusset plate and can move laterally. Therefore, the effective length is given as K = 1.2 by AISC *Design Guide 24*. The unbraced length (L_c) of the gusset plate is taken along the center line of the brace. The Whitmore section and unbraced length are shown in Figure 3, in accordance with Figure 12 Case A of Thornton and Lini (2011).
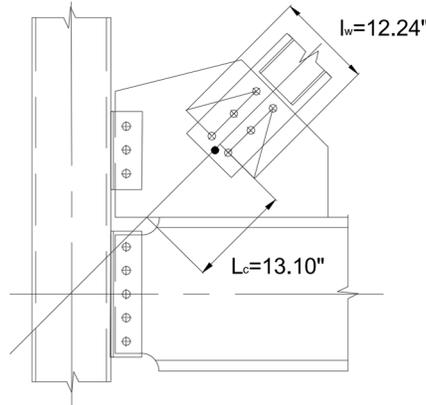

Fig. 3. Geometry of the gusset plate Whitmore section and unbraced length

$$\frac{KL_c}{r_c} = \frac{1.2(13.10 \text{ in.})}{r_c}$$

$$r_c = \frac{t_p}{\sqrt{12}}$$

$$= \frac{3/4 \text{ in.}}{\sqrt{12}}$$

$$= 0.216 \text{ in.}$$

$$\frac{KL_c}{r_c} = \frac{1.2(13.10 \text{ in.})}{0.216 \text{ in.}}$$

$$= 72.8$$

Using AISC *Manual* Table 4-14 for F_y = 50 ksi and the calculated Kl/r, the available nominal stress is:

$\phi F_n = 30.6 \text{ ksi}$

The available flexural buckling strength of a gusset plate in compression is determined from AISC *Specification* Section E3.

$\phi P_n = \phi F_n A_g$ (from *Spec*. Eq. E3-1)

$\quad = (30.6 \text{ ksi})(12.24 \text{ in.})(3/4 \text{ in.})$

$\quad = 280.9 \text{ kips}$

The eccentricity of the splice plate with respect to the gusset plate is:

$$e = \frac{t_s + t_p}{2}$$

$$= \frac{(7/8 \text{ in.}) + (3/4 \text{ in.})}{2}$$

$$= 0.813 \text{ in.}$$

Therefore, the moment acting on the gusset plate is:

$$M_r = \frac{P_r e}{2}$$

$$= \frac{(130 \text{ kips})(0.813 \text{ in.})}{2}$$

$$= 52.8 \text{ kip-in.}$$

The available strength of a gusset plate in flexure is determined from AISC *Specification* Section F11.

$$\phi M_n = \phi F_y Z \hfill \text{(from } \textit{Spec.} \text{ Eq. F11-1)}$$

$$Z = \frac{l_w t_p^2}{4}$$

$$= \frac{(12.24 \text{ in.})(3/4\text{in.})^2}{4}$$

$$= 1.72 \text{ in.}^3$$

$$\phi M_n = 0.9(50 \text{ ksi})(1.72 \text{ in.}^3)$$

$$= 77.5 \text{ kip-in.}$$

From AISC *Design Guide 24* the interaction of the axial force and moment acting on the splice plate is expressed as:

$$\left(\frac{P_r}{\phi P_n}\right)^2 + \frac{M_r}{\phi M_n} = \left(\frac{130 \text{ kips}}{280.9 \text{ kips}}\right)^2 + \frac{52.8 \text{ kip-in}}{77.5 \text{ kip-in}} = 0.90 \leq 1.0 \hfill \text{(from } \textit{DG 24} \text{ Eq. 7-8)}$$

"Buckling" of the splice plate

HSS connections with lapped splice plates are susceptible to forming plastic hinges in the gusset plate and splice plate on either side of the overlap, creating a failure mechanism (Tremblay & Davaran, 2020). Therefore, the moment demand due to the splice-to-gusset plate eccentricity must be considered in the splice plate.

$$\phi P_n = \phi F_n A_g \hfill \text{(from } \textit{Spec.} \text{ Eq. E3-1)}$$

$$= 0.9(50 \text{ ksi})(8 \text{ in.})(7/8 \text{ in.})$$

$$= 315 \text{ kips}$$

$$\phi M_n = \phi F_y Z = \phi F_y \frac{b_s t_s^2}{4} \hfill \text{(from } \textit{Spec.} \text{ Eq. F11-1)}$$

$$\phi M_n = 0.9(50 \text{ ksi})\frac{(8 \text{ in.})(7/8 \text{ in.})^2}{4}$$

$$= 68.9 \text{ kip-in}$$

From AISC *Design Guide 24* the interaction of the axial force and moment acting on the splice plate is expressed as:

$$\left(\frac{P_r}{\phi P_n}\right)^2 + \frac{M_r}{\phi M_n} = \left(\frac{130 \text{ kips}}{315 \text{ kips}}\right)^2 + \frac{52.8 \text{ kip-in}}{68.9 \text{ kip-in}} = 0.94 \leq 1.0 \hfill \text{(from } \textit{DG 24} \text{ Eq. 7-8)}$$

Strength of the fillet welds between the gusset plate and the wide-flange

For the gusset plate to be in equilibrium, V_{rb} and H_{rb} act at the gusset plate to wide-flange interface at $\alpha=13.25$ in. from the column face. The 10 in. weld is centered about this point, therefore the resultant force acts with no eccentricity to the weld. From AISC *Specification* Section J2.4, the available weld strength, per unit length, is:

$$\phi R_n = \phi 0.60 F_{EXX} \frac{w}{\sqrt{2}} l(2 \text{ lines}) \hfill \text{(from } \textit{Spec.} \text{ Eq J2-4)}$$

$$= 0.75(0.60)(70 \text{ ksi})\frac{1/4}{\sqrt{2}}(10.00 \text{ in.})(2 \text{ lines}) \text{ for a 1/4 in.}$$

weld

$$= 111.4 \text{ kips} > \sqrt{(49.0 \text{ kips})^2 + (66.8 \text{kips})^2} = 82.8 \text{ kips}$$

o.k.

From the AISC *Specifications* Table J2.4 the minimum weld size is 3/16 in. Therefore, a 1/4 in. size is sufficient.

Tensile yielding of the coped wide-flange web

The available strength of the coped wide-flange web for the limit state of tensile yielding is given by AISC *Specification* Section J4.1(a).

$$\phi R_n = \phi F_y A_g \hfill \text{(from } \textit{Spec.} \text{ Eq. J4-1)}$$

$$= 0.90(50 \text{ ksi})(15 \text{ in.})(0.730 \text{ in.})$$

$$= 492.8 \text{ kips} > 25.1 \text{ kips} \quad \textbf{o.k.}$$

Tensile rupture of the coped wide-flange web

The available strength of the coped wide-flange web for the limit state of tensile rupture is given by AISC *Specification* Section J4.1(b).

$$\phi R_n = \phi F_u A_e \qquad \text{(from *Spec*. Eq. J4-2)}$$

Where A_e is defined as $A_n U$ by AISC *Specification* Section D3, and U is equal to 1.0 (*Spec*. Table D3. 1) for a load transmitted directly into the cross-section by fasteners.

$$A_n = A_g - 5(d_h + 1/16 \text{ in.})t_p$$
$$= (15 \text{ in.})(0.730 \text{ in.}) - 5(15/16 \text{ in.} + 1/16 \text{ in.})(0.730 \text{in.}) \qquad (d_h \text{ from *Spec*. B3.3(b) and Table J3.3)}$$
$$= 7.30 \text{ in.}^2$$
$$A_e = A_n U \qquad \text{(from *Spec*. Eq. D3-1)}$$
$$= 7.30 \text{ in.}^2$$
$$\phi R_n = 0.75(65 \text{ ksi})(7.30 \text{ in.}^2)$$
$$= 355.9 \text{ kips} > 25.1 \text{ kips} \quad \textbf{o.k.}$$

Shear yielding of the coped wide-flange web

The available strength of the coped wide-flange web for the limit state of shear yielding at the connection is given by AISC *Specification* Section J4.2(a). The coped wide-flange web must resist the V_{rb} and the beam shear.

$$\phi R_n = \phi 0.60 F_y A_{gv} \qquad \text{(from *Spec*. Eq. J4-3)}$$
$$= 1.00(0.60)(50 \text{ ksi})(15 \text{ in.})(0.730 \text{ in.})$$
$$= 328.5 \text{ kips} > 209.0 \text{ kips} \quad \textbf{o.k.}$$

Shear rupture of the coped wide-flange web

The available strength of the coped wide-flange web for the limit state of shear rupture at the connection is given by AISC *Specification* Section J4.2(b). The coped wide-flange web must resist the V_{rb} and the beam shear.

$$\phi R_n = \phi 0.60 F_u A_{nv} \qquad \text{(from *Spec*. Eq. J4-4)}$$
$$= 0.75(0.60)(65 \text{ ksi})[15 \text{ in.} - 5(15/16 \text{ in.} + 1/16 \text{ in.})](0.730 \text{ in.})$$
$$= 213.5 \text{ kips} > 209.0 \text{ kips} \quad \textbf{o.k.}$$

Strength of the bolts between the clip angles and gusset plate

The available shear strength of a 7/8 in. Group 150 bolt (condition N) subject to double shear is given as 61.3 kips per bolt in Table 7-1 of the AISC *Manual*.

The available strength of the bolts in bearing is given by AISC Specification Section J3.11, assuming deformation at service load is a design consideration.

$$\phi R_n = \phi 2.4 dt F_u \qquad \text{(from *Spec*. Eq. J3-6a)}$$
$$= 0.75(2.4)(7/8 \text{ in.})(2)(3/8 \text{ in.})(65 \text{ ksi})$$
$$= 76.7 \text{ kips/bolt}$$

The available tearout strength of the clip angles at the interior bolt due to vertical force components is determined from AISC *Specification* Section J3.11.

$$\phi R_n = \phi 1.2 l_c t F_u \qquad \text{(from *Spec*. Eq. J3-6c)}$$
$$l_c = s - d_h$$
$$= 3 \text{ in.} - 15/16 \text{ in.}$$
$$= 2.06 \text{ in.}$$
$$\phi R_n = 0.75(1.2)(2.06 \text{ in.})(2)(3/8 \text{ in.})(65 \text{ ksi})$$
$$= 90.5 \text{ kips/bolt}$$

The available tearout strength of the clip angles at the exterior bolt due to vertical force components is determined from AISC *Specification* Section J3.11.

$$\phi R_n = \phi 1.2 l_c t F_u \qquad \text{(from *Spec*. Eq. J3-6c)}$$
$$l_c = l_e - d_h/2$$

$$= 2 \text{ in.} - (15/16 \text{ in.})/2$$
$$= 1.53 \text{ in.}$$
$$\phi R_n = 0.75(1.2)(1.53 \text{ in.})(2)(3/8 \text{ in.})(65 \text{ ksi})$$
$$= 66.2 \text{ kips/bolt}$$

Therefore, the strength of this bolt group is governed by bolt shear. The 3 bolts connecting the clip angles to the gusset plate must resist V_{rc} acting at the eccentricity of e_x from the column face, and H_{rc} acting at the bolt centroid.
$$e_x = 2 \text{ in.}$$

From the AISC *Manual* Table 7-6, the effective number of bolts (C) for a 3-bolt group with an eccentricity of 2.0 in. and an angle of 0° relative to the force is 2.23.

The total utilization of this bolt group is:
$$\frac{42.9 \text{ kips}}{(2.23 \text{ bolts})(61.3 \text{ kips/bolt})} + \frac{25.1 \text{ kips}}{(3 \text{ bolts})(61.3 \text{ kips/bolt})} = 0.45 < 1.0 \quad \textbf{o.k.}$$

Bolt spacing requirements are satisfied per the *Strength of the bolts between the splice plate and gusset plate* check.

Strength of the bolts between the clip angles and wide-flange section web

As per the *Strength of the bolts between the clip angles and gusset plate* check the strength of this bolt group is governed by bolt shear, except for the exterior bolt. The exterior bolt may be governed by bolt tearout:
$$\phi R_n = \phi 1.2 l_c t F_u \qquad \text{(from \textit{Spec.} Eq. J3-6c)}$$
$$l_c = l_e - d_h/2$$
$$= 1.5 \text{ in.} - (15/16 \text{ in.})/2$$
$$= 1.03 \text{ in.}$$
$$\phi R_n = 0.75(1.2)(1.03 \text{ in.})(0.730 \text{ in.})(65 \text{ ksi})$$
$$= 44.0 \text{ kips/bolt}$$

The 5 bolts between the clip angles and wide-flange web must resist the resultant load from V_{rb} and H_{rb}, the 160 kip beam shear force, and the 91.9 kip beam axial force. V_{rb} acts at an eccentricity of $e_x = 2.0$ in. from the bolt group. The remaining forces act on the bolt centroid with the following magnitude:
$$V_f = \sqrt{(160 \text{ kips})^2 + (25.1 \text{ kips})^2}$$
$$= 161.9 \text{ kips}$$

From the AISC *Manual* Table 7-6, the effective number of bolts (C) for a 5-bolt group with an eccentricity of 2.0 in. with an angle of 0° relative to the force is 4.39. Therefore, the total utilization of this bolt group is:
$$\frac{49.0 \text{ kips}}{(4.39 \text{ bolts})(61.3 \text{ kips/bolt})} + \frac{161.9 \text{ kips}}{(4 \text{ bolts})(61.3 \text{ kips/bolt}) + (1 \text{ bolt})(44.0 \text{ kips/bolt})} = 0.74 < 1.0 \quad \textbf{o.k.}$$

Bolt spacing requirements are satisfied per the *Strength of the bolts between the splice plate and gusset plate* check.

Shear yielding of the clip angles

The available strength of the four clip angles for the limit state of shear yielding is given by AISC *Specification* Section J4.2(a).
$$\phi R_n = \phi 0.60 F_y A_{gv} \qquad \text{(from \textit{Spec.} Eq. J4-3)}$$

The angles between the column and gusset plate must resist V_{rc}:
$$A_{gv} = 2 L_a t_a$$
$$= 2(10 \text{ in.})(3/8 \text{ in.})$$
$$= 7.5 \text{ in.}^2$$
$$\phi R_n = 1.00(0.60)(50 \text{ ksi})(7.5 \text{ in.}^2)$$
$$= 225 \text{ kips} > 42.9 \text{ kips} \quad \textbf{o.k.}$$

The angles between the column and beam web must resist the beam shear and V_{rb}:
$$A_{gv} = 2 L_a t_a$$
$$= 2(16 \text{ in.})(3/8 \text{ in.})$$
$$= 12.0 \text{ in.}^2$$
$$\phi R_n = 1.00(0.60)(50 \text{ ksi})(12.0 \text{ in.}^2)$$

$= 360.0$ kips > 209.0 kips   **o.k.**

Shear rupture of the clip angles

The available strength of the four clip angles for the limit state of shear rupture is given by AISC *Specification* Section J4.2(b).

$$\phi R_n = \phi 0.60 F_u A_{nv} \qquad \text{(from \textit{Spec.} Eq. J4-4)}$$

The angles between the column and gusset plate must resist V_{rc}:

$$A_{nv} = A_{gv} - 3(15/16 \text{ in.} + 1/16 \text{ in.})(2)(3/8 \text{ in.})$$
$$= 5.25 \text{ in.}^2$$
$$\phi R_n = 0.75(0.60)(65 \text{ ksi})(5.25 \text{ in.}^2)$$
$$= 153.6 \text{ kips} > 42.9 \text{ kips} \quad \textbf{o.k.}$$

The angles between the column and beam web must resist the beam shear and V_{rb}:

$$A_{nv} = A_{gv} - 5(15/16 \text{ in.} + 1/16 \text{ in.})(2)(3/8 \text{ in.})$$
$$= 8.25 \text{ in.}^2$$
$$\phi R_n = 0.75(0.60)(65 \text{ ksi})(8.25 \text{ in.}^2)$$
$$= 241.3 \text{ kips} > 209.0 \text{ kips} \quad \textbf{o.k.}$$

Block shear of the clip angles

The available strength of the four clip angles for the limit state of block shear is given by AISC *Specification* Section J4.3 where U_{bs} is 1.0 for a uniform load. The block shear pattern is shown in Figure 4-a.

$$\phi R_n = \phi 0.60 F_u A_{nv} + \phi U_{bs} F_u A_{nt} \le \phi 0.60 F_y A_{gv} + \phi U_{bs} F_u A_{nt} \qquad \text{(from \textit{Spec.} Eq. J4-5)}$$
$$A_{gv} = 2(16 \text{ in.} - 2 \text{ in.})(3/8 \text{ in.}) + 2(10 \text{ in.} - 2 \text{ in.})(3/8 \text{ in.})$$
$$= 16.50 \text{ in.}^2.$$
$$A_{nv} = A_{gv} - [8 - 2(0.5)](15/16 \text{ in.} + 1/16 \text{ in.})(2)(3/8 \text{ in.})$$
$$= 11.25 \text{ in.}^2.$$
$$A_{nt} = 2[(2 \text{ in.})2(3/8 \text{ in.}) - 0.5(15/16 \text{ in.} + 1/16 \text{ in.})2(3/8 \text{ in.})]$$
$$= 2.25 \text{ in.}^2.$$
$$\phi R_n = 0.75(0.60)(65 \text{ ksi})(11.25 \text{ in.}^2.) + 0.75(1.0)(65 \text{ ksi})(2.25 \text{ in.}^2.)$$
$$\le 0.75(0.60)(50 \text{ ksi})(16.5 \text{ in.}^2.) + 0.75(1.0)(65 \text{ ksi})(2.25 \text{ in.}^2.)$$
$$= 438.8 \text{ kips} \le 480.9 \text{ kips}$$
$$= 438.8 \text{ kips} > 251.9 \text{ kips} \quad \textbf{o.k.}$$

Block shear of the beam web and gusset plate

The available strength of the beam web and gusset plate for the limit state of block shear is given by AISC *Specification* Section J4.3 where U_{bs} is 1.0 for a uniform load. The block shear pattern is shown in Figure 4-b.

$$\phi R_n = \phi 0.60 F_u A_{nv} + \phi U_{bs} F_u A_{nt} \le \phi 0.60 F_y A_{gv} + \phi U_{bs} F_u A_{nt} \qquad \text{(from \textit{Spec.} Eq. J4-5)}$$
$$A_{gv} = (15 \text{ in.} - 1.5 \text{ in.})(0.730 \text{ in.}) + (10\text{-}13/16)(3/4 \text{ in.})$$
$$= 18.0 \text{ in.}^2.$$
$$A_{nv} = A_{gv} - 4.5(15/16 \text{ in.} + 1/16 \text{ in.})(0.730 \text{ in.}) - 2.5(15/16 \text{ in.} + 1/16 \text{ in.})(3/4 \text{ in.})$$
$$= 12.8 \text{ in.}^2.$$
$$A_{nt} = [1.5 \text{ in.} - 0.5(15/16 \text{ in.} + 1/16 \text{ in.})](0.730 \text{ in.}) + [1.5 \text{ in.} - 0.5(15/16 \text{ in.} + 1/16 \text{ in.})](3/4 \text{ in.})$$
$$= 1.48 \text{ in.}^2.$$
$$\phi R_n = 0.75(0.60)(65 \text{ ksi})(12.8 \text{ in.}^2.) + 0.75(1.0)(65 \text{ ksi})(1.48 \text{ in.}^2.)$$
$$\le 0.75(0.60)(50 \text{ ksi})(18.0 \text{ in.}^2.) + 0.75(1.0)(65 \text{ ksi})(1.48 \text{ in.}^2.)$$
$$= 446.6 \text{ kips} \le 477.2 \text{ kips}$$
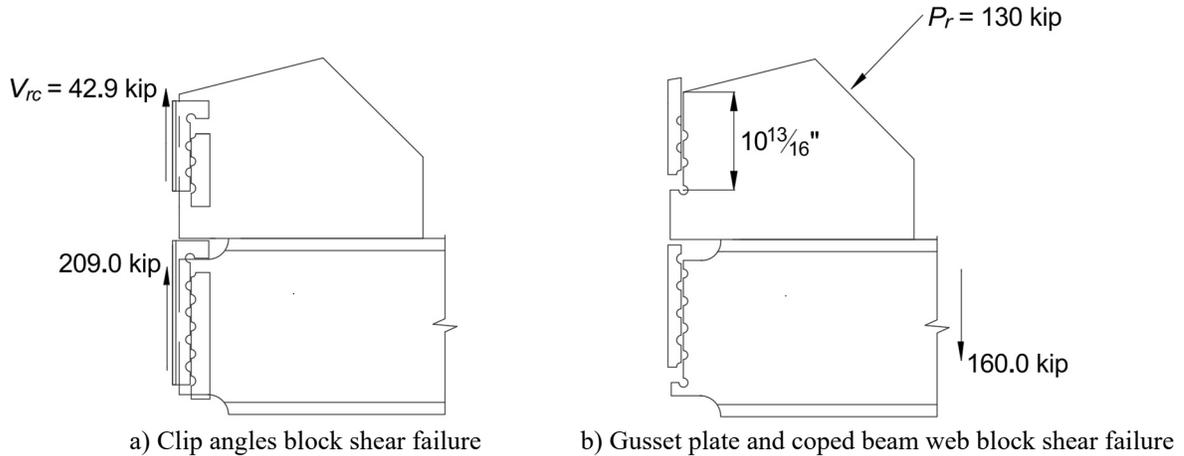$$= 446.6 \text{ kips} > 251.9 \text{ kips} \quad \textbf{o.k.}$$

a) Clip angles block shear failure   b) Gusset plate and coped beam web block shear failure

Fig. 4. Block shear failure patterns of the clip angles (a) and wide-flange section web and gusset plate (b)

Shear of the weld metal at the column

The welds between the four clip angles and the column are at an eccentricity (e_x) to the bolt group centerline:

$e_x = 2$ in.

The two clip angles between the gusset plate and column must resist V_{rc} and H_{rc} ($L_w = 10$ in.). From the AISC *Manual* Table 8-4, *Coefficients, C, for Eccentrically Loaded Weld Groups (Angle = 0°)*, C=3.51. The weld size is 3/16 in. which means $D = 3$ (number of sixteenths-of-an-inch in the fillet weld size).

$$\phi R_{n,br} = \phi CDL_{w,br} \hspace{3cm} \text{(from \textit{AISC Manual}. Table 8-4)}$$
$$= 0.75(3.51)(3)(10) \text{ kips}$$
$$= 79.0 \text{ kips} > \sqrt{(42.9 \text{ kips})^2 + (25.1 \text{ kips})^2} = 49.7 \text{ kips} \quad \textbf{o.k.}$$

The two clip angles between the beam web and column must resist V_{rb}, H_{rc}, and the beam shear ($L_w = 16$ in.). From the AISC *Manual* Table 8-4, *Coefficients, C, for Eccentrically Loaded Weld Groups (Angle = 0°)*, C=3.70. The weld size is 5/16 in. which means $D = 5$.

$$\phi R_{n,be} = \phi CDL_{w,be} \hspace{3cm} \text{(from \textit{AISC Manual}. Table 8-4)}$$
$$= 0.75(3.70)(5)(16) \text{ kips}$$
$$= 222.0 \text{ kips} > \sqrt{(49.0 \text{ kips} + 160 \text{ kips})^2 + (25.1 \text{ kips})^2} = 210.5 \text{ kips} \quad \textbf{o.k.}$$

From the AISC *Specifications* Table J2.4 the minimum weld size is 3/16 in. Therefore, both weld sizes are sufficient.

Plastification of the HSS column sidewall

The available strength of the HSS column connecting face can be obtained from ASIC *Design Guide 24*, Table 9-2, by considering the footprint of two angles as being equivalent to the footprint of a rectangular branch in an HSS-to-HSS T-connection. Per Table 9-2A, only Limit State 1: Chord Plastification, applies, since $\beta = 0.675$.

$$\beta = \text{branch width / column width}$$
$$= [2(3 \text{ in.}) + 3/4 \text{ in.}] / 10 \text{ in.}$$
$$= 0.675$$

$$\phi R_n = \phi F_y t^2 \left(\frac{2(L_a/B)}{1-\beta} + \frac{4}{\sqrt{1-\beta}} \right) Q_f \hspace{2cm} \text{(from \textit{DG 24}. Eq. 2-2)}$$

$$Q_f = 0.4 \leq \left[1.3 - 0.4 \frac{U}{\beta} \right] \leq 1.0 \hspace{2cm} \text{(from \textit{Spec}. Eq. K1-4)}$$

$$U = \left[\frac{P_u}{F_y A_g} + \frac{M_u}{F_y S} \right] \hspace{2cm} \text{(from \textit{Spec}. Eq. K1-6)}$$

$$= \left[\frac{-501.9 \text{ kips}}{(50 \text{ ksi})(17.2 \text{ in.}^2)} + 0 \right]$$

$$= 0.584$$

9

$$Q_f = 0.4 \leq \left[1.3 - 0.4 \frac{0.584}{0.675}\right] \leq 1.0$$
$$= 0.95$$

The two clip angles between the gusset plate and the column must resist H_{rc}.
$$\phi R_n = 1.00(50 \text{ ksi})(0.465)^2 \left(\frac{2(10 \text{ in.})/(10 \text{ in.})}{1 - 0.675} + \frac{4}{\sqrt{1 - 0.675}}\right) 0.95$$
$$= 98.2 \text{ kips} > 25.1 \text{ kips} \quad \textbf{o.k.}$$

The two clip angles between the beam web and the column must resist H_{rc}.
$$\phi R_n = 1.00(50 \text{ ksi})(0.465)^2 \left(\frac{2(16 \text{ in.})/(10 \text{ in.})}{1 - 0.675} + \frac{4}{\sqrt{1 - 0.675}}\right) 0.95$$
$$= 173.2 \text{ kips} > 25.1 \text{ kips} \quad \textbf{o.k.}$$

Limits of Applicability (from *DG 24*. Table. 9-2A):

    HSS wall slenderness: B/t or $H/t = 10 \text{ in.} / 0.464 \text{ in.} = 22 \leq 40$ **o.k.**

    Material strength: $F_y = 50 \text{ ksi} \leq 52 \text{ ksi}$ **o.k.**

    Ductility: ASTM A500/A500M Grade C is acceptable

    Assume the end distance is greater than $B\sqrt{1 - \beta} = (10 \text{ in.})\sqrt{1 - 0.675} = 5.7 \text{ in.}$

a) Connection geometry and the gusset plate centroid   b) Gusset plate, beam, and column free-body diagrams

Fig. 5. Geometry (a) and individual component free-body diagrams (b) for a tension brace scenario

The following limit states must also be checked if the brace is in tension:

Splice plate tensile yielding

The available strength of the slotted splice plate for the limit state of tensile yielding is given by AISC *Specification* Section J4.1(a).

$$\phi R_n = \phi F_y A_g \qquad \text{(from \textit{Spec}. Eq. J4-1)}$$
$$= 0.90(50 \text{ ksi})(8 \text{ in.})(7/8 \text{ in.})$$
$$= 315.0 \text{ kips} > 130 \text{ kips} \quad \textbf{o.k.}$$

Splice plate tensile rupture (of whole cross section)

The available strength of the slotted splice plate for the limit state of tensile rupture is given by AISC *Specification* Section J4.1(b).

$$\phi R_n = \phi F_u A_e \qquad \text{(from \textit{Spec}. Eq. J4-2)}$$

where A_e is defined as $A_n U$ by AISC Specification Section D3, and U is equal to 1.0 (Spec. Table D3. 1) for a load transmitted directly into the cross-section by fasteners.

$$A_n = A_g - 2(d_h + 1/16 \text{ in.}) t_p \qquad (d_h \text{ from Spec. J3.3(b) and Table J3.3)}$$
$$= (8 \text{ in.})(7/8 \text{ in.}) - 2(15/16 \text{ in.} + 1/16 \text{ in.})(7/8 \text{ in.})$$
$$= 5.25 \text{ in.}^2$$
$$A_e = A_n U \qquad \text{(from Spec. Eq. D3-1)}$$
$$= 5.25 \text{ in.}^2$$
$$\phi R_n = 0.75(65 \text{ ksi})(5.25 \text{ in.}^2)$$
$$= 255.9 \text{ kips} > 130 \text{ kips} \quad \textbf{o.k.}$$

Splice plate block shear failure

The available strength of the splice plate for the limit state of block shear is given by AISC *Specification* Section J4.3.

1. One viable failure pattern consists of a shear failure along the welds and a tensile failure between the edge of the brace and edge of the plate where U_{bs} is 1.0 for a uniform load.

$$\phi R_n = \phi 0.60 F_u A_{nv} + \phi U_{bs} F_u A_{nt} \leq \phi 0.60 F_y A_{gv} + \phi U_{bs} F_u A_{nt} \qquad \text{(from \textit{Spec}. Eq. J4-5)}$$
$$A_{gv} = 2(6 \text{ in.})(7/8 \text{ in.})$$
$$= 10.5 \text{ in}^2.$$
$$A_{nv} = A_{gv}$$
$$A_{nt} = (8 \text{ in.} - 6 \text{ in.})(7/8 \text{ in.})$$
$$= 1.75 \text{ in}^2.$$
$$\phi R_n = 0.75(0.60)(65 \text{ ksi})(10.5 \text{ in}^2.) + 0.75(1.0)(65 \text{ ksi})(1.75 \text{ in}^2.)$$
$$\leq 0.75(0.60)(50 \text{ ksi})(10.5 \text{ in}^2.) + 0.75(1.0)(65 \text{ ksi})(1.75 \text{ in}^2.)$$
$$= 392.4 \text{ kips} \leq 321.6 \text{ kips}$$
$$= 183.7 \text{ kips} > 130 \text{ kips} \quad \textbf{o.k.}$$

2. Another viable failure pattern consists of a shear failure between the 6 bolt holes and the plate end, with a tensile failure between one pair of bolts where U_{bs} is 1.0 for a uniform load.

$$\phi R_n = \phi 0.60 F_u A_{nv} + \phi U_{bs} F_u A_{nt} \leq \phi 0.60 F_y A_{gv} + \phi U_{bs} F_u A_{nt} \qquad \text{(from \textit{Spec}. Eq. J4-5)}$$
$$A_{gv} = 2(4 \text{ in.} + 4 \text{ in.} + 2 \text{ in.})(7/8 \text{ in.})$$
$$= 17.5 \text{ in}^2.$$
$$A_{nv} = A_{gv} - 2(2.5)(15/16 \text{ in.} + 1/16 \text{ in.})(7/8 \text{ in.})$$
$$= 13.12 \text{ in}^2.$$
$$A_{nt} = (3 \text{ in.})(7/8 \text{ in.}) - (15/16 \text{ in.} + 1/16 \text{ in.})(7/8 \text{ in.})$$
$$= 1.75 \text{ in}^2.$$

Since A_{gv}, A_{nv}, and A_{nt} of failure pattern 2 are greater than or equal to those of failure pattern 1, it is not critical.

Gusset plate block shear failure

The available strength of the gusset plate for the limit state of block shear is given by AISC *Specification* Section J4.3 where U_{bs} is 1.0 for a uniform load.

$$\phi R_n = \phi 0.60 F_u A_{nv} + \phi U_{bs} F_u A_{nt} \le \phi 0.60 F_y A_{gv} + \phi U_{bs} F_u A_{nt} \qquad \text{(from \emph{Spec.} Eq. J4-5)}$$

$$A_{gv} = 2(4 \text{ in.} + 4 \text{ in.} + 1.5 \text{ in.})(3/4 \text{ in.})$$
$$= 14.3 \text{ in}^2.$$
$$A_{nv} = A_{gv} - 2(2.5)(15/16 \text{ in.} + 1/16 \text{ in.})(3/4 \text{ in.})$$
$$= 10.5 \text{ in}^2.$$
$$A_{nt} = (3 \text{ in.})(3/4 \text{ in.}) - (15/16 \text{ in.} + 1/16 \text{ in.})(3/4 \text{ in.})$$
$$= 1.5 \text{ in}^2.$$
$$\phi R_n = 0.75(0.60)(65 \text{ ksi})(10.5 \text{ in}^2.) + 0.75(1.0)(65 \text{ ksi})(1.5 \text{ in}^2.)$$
$$\le 0.75(0.60)(50 \text{ ksi})(14.3 \text{ in}^2.) + 0.75(1.0)(65 \text{ ksi})(1.5 \text{ in}^2.)$$
$$= 380.2 \text{ kips} \le 394.9 \text{ kips}$$
$$= 380.2 \text{ kips} > 130 \text{ kips} \quad \textbf{o.k.}$$

HSS brace tensile yielding

The available strength of the HSS brace for the limit state of tensile yielding is given by AISC *Specification* Section D2.(a).

$$\phi P_n = \phi F_y A_g \qquad \text{(from \emph{Spec.} Eq. D2-1)}$$
$$= 0.90(50 \text{ ksi})(7.58 \text{ in.}^2)$$
$$= 341.1 \text{ kips} > 130 \text{ kips} \quad \textbf{o.k.}$$

HSS brace tensile rupture (circumferential fracture of the whole cross section)

The available strength of the HSS brace for the limit state of tensile rupture is given by AISC *Specification* Section D2.(b), where A_e is defined as $A_n U$ by AISC Specification Section D3, and U is defined in Spec. Table D3. 1.

$$\bar{x} = b - \frac{2b^2 + tH - 2t^2}{2H + 4b - 4t} \qquad \text{(from \emph{Spec.} Table D3. 1.)}$$

$$b = \frac{B}{2} - \frac{t_p + 1/16 \text{ in.}}{2} \qquad \text{(The HSS slot is 1/16 in. wider than the splice plate thickness)}$$
$$= \frac{6 \text{ in.}}{2} - \frac{7/8 \text{ in.} + 1/16 \text{ in.}}{2}$$
$$= 2.53 \text{ in.}$$
$$\bar{x} = 2.53 \text{ in.} - \frac{2(2.53 \text{ in.})^2 + (0.349 \text{ in.})(6 \text{ in.}) - 2(0.349 \text{ in.})^2}{2(6 \text{ in.}) + 4(2.53 \text{ in.}) - 4(0.349 \text{ in.})}$$
$$= 1.82 \text{ in.}$$
$$U = 1 - \frac{\bar{x}}{l} \qquad \text{(from Spec. Table D3. 1.)}$$
$$= 1 - \frac{1.82 \text{ in.}}{6 \text{ in.}}$$
$$= 0.697$$
$$A_n = A_g - 2(t_p + 1/16 \text{ in.})t$$
$$= (7.58 \text{ in.}^2) - 2(7/8 \text{ in.} + 1/16 \text{ in.})(0.349 \text{ in.})$$
$$= 6.92 \text{ in.}^2$$
$$A_e = A_n U \qquad \text{(from Spec. Eq. D3-1)}$$
$$= (6.92 \text{ in.}^2)(0.697)$$
$$= 4.83 \text{ in.}^2$$
$$\phi P_n = \phi F_u A_e \qquad \text{(from \emph{Spec.} Eq. D2-2)}$$
$$\phi P_n = 0.75(62 \text{ ksi})(4.83 \text{ in.}^2)$$
$$= 224.6 \text{ kips} > 130 \text{ kips} \quad \textbf{o.k.}$$

HSS brace block shear failure (tear out beside the fillet welds)

The available strength of the HSS brace for the limit state of block shear is given by AISC *Specification* Section J4.3 where U_{bs} is 1.0 for a uniform load.

$$\phi R_n = \phi 0.60 F_u A_{nv} + \phi U_{bs} F_u A_{nt} \leq \phi 0.60 F_y A_{gv} + \phi U_{bs} F_u A_{nt} \qquad \text{(from } Spec. \text{ Eq. J4-5)}$$

$$A_{gv} = 4(6 \text{ in.})(0.349 \text{ in.})$$
$$\quad = 8.38 \text{ in}^2.$$
$$A_{nv} = A_{gv}$$
$$A_{nt} = 0 \text{ (assumed no welding is performed at the end of the splice plate)}$$
$$\phi R_n = 0.75(0.60)(62 \text{ ksi})(8.38 \text{ in}^2) \leq 0.75(0.60)(50 \text{ ksi})(8.38 \text{ in}^2.)$$
$$\quad = 233.8 \text{ kips} \leq 188.6 \text{ kips}$$
$$\quad = 188.6 \text{ kips} > 130 \text{ kips} \quad \textbf{o.k.}$$

The only force which changes in magnitude from Figure 2 to Figure 5 due to the reversal of the brace axial force is the vertical reaction on the bottom of the column. The column compressive force decreases and therefore has a conservative effect on the chord stress function (*U*) in the *Plastification of the HSS column sidewall* limit state.

References:

AISC. (2022). *Specification for Structural Steel Buildings*, AISC 360-22, American Institute of Steel Construction, Chicago, IL.

AISC. (2023). *Steel Construction Manual*, 16th. Edition, American Institute of Steel Construction, Chicago, IL.

Dranger, S. T., & Thornton, A. W. (2025). *A Derivation of the Uniform Force Method for Analysis and Design of Gusset Plate Connections for Vertical Diagonal Bracing*, Engineering Journal, 2nd. quarter, American Institute of Steel Construction, Chicago, IL.

Packer, J. A., & Olson, K. (2024). *Hollow Structural Section Connections*, AISC Design Guide No. 24, 2nd. edition, American Institute of Steel Construction, Chicago, IL.

Thornton, W.A., & Lini, C. (2011). *The Whitmore Section – How to Use the Whitmore Method for Tension and Compression Strength Checks*, Modern Steel Construction, July Issue, American Institute of Steel Construction, Chicago, IL.

Tremblay, R., & Davaran, A. (2020). *Design of Bolted Single Shear Lap Connection with Different Thicknesses of Splice Plates*, Journal of Structural Engineering, March Issue, American Society of Civil Engineers, Reston, VA.

October 2025